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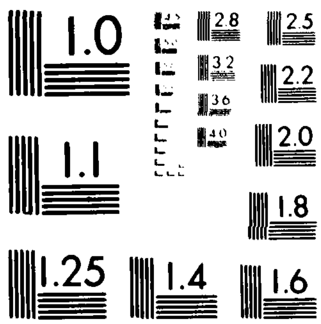

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RESEARCH AND DEVELOPMENT TECHNICAL REPORT

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EFFECT OF VARIOUS DIELECTRICS ON THE DESIGN OF MILLIMETER-WAVE LINE SCANNING ANTENNAS

Kenneth L. Klohn
ELECTRONICS TECHNOLOGY & DEVICES LABORATORY

November 1979

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1. REPORT NUMBER: 14 DELET-TR-79-24
2. GOVT ACCESSION NO.: AD-A083 791
3. RECIPIENT'S CATALOG NUMBER: Research and Development
4. TITLE (and Subtitle): 6 Effect of Various Dielectrics on the Design of Millimeter-Wave Line Scanning Antennas.
5. TYPE OF REPORT & PERIOD COVERED: Technical Report.
6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): 10 Kenneth L. Klohn
8. CONTRACT OR GRANT NUMBER(s): N/A
9. PERFORMING ORGANIZATION NAME AND ADDRESS: US Army Electronics Research and Development Command (ERADCOM) ATTN: DELET-MJ, Fort Monmouth, NJ 07703
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 16 1L162705AH94 07 17 07
11. CONTROLLING OFFICE NAME AND ADDRESS
12. REPORT DATE: 11 November 1979
13. NUMBER OF PAGES: 20
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): 12 26
15. SECURITY CLASS. (of this report): UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release: Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

EPSILON EPSILON EPSILON

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Dielectric Waveguide
Millimeter Waves
Guide Wavelengths
Perturbation Spacing
Radiation Angle
Angular Scan
Line Scanning Antenna

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The effect of using various materials having different relative dielectric constants on the theoretical design of millimeter-wave line scanning antennas has been examined. Three typical dielectrics were chosen for comparative purposes; silicon (ϵ_r = 12), sapphire (ϵ_r = 9.4), and boron nitride (ϵ_r = 4). Theoretical calculations were made for each material over the frequency range of 60 to 220 GHz, with special emphasis at 94 GHz due to its increasing use in new millimeter-wave systems. Comparisons were made among the dielectric materials in regard to their maximum allowable physical size for single E^y_{11}

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mode operation. Using each of the three dielectrics with periodic surface perturbations as a line scanning antenna, the effects on the angle of radiating energy due to changes in the waveguide size and perturbation spacing were determined and evaluated.

CONTENTS

INTRODUCTION

Recent experimental work has investigated the use of dielectric materials with periodic surface perturbations for millimeter-wave antennas.[1-3] The use of dielectrics becomes more attractive as frequency increases because they have lower losses than metal waveguide or microstrip. In addition, the tolerances required to build dielectric antennas are much less severe than metal guides or microstrip and thus opens the possibility for batch processing. This would reduce fabrication costs and satisfy one of the main objectives of millimeter-wave research and development, namely, to provide affordable systems.

The design of dielectric waveguides and antennas, i.e., width a and height b, at any given frequency is influenced by the relative dielectric constant of the material. Theoretical calculations were made based on Marcatili's equations[4] to establish the maximum dimensions of the dielectric waveguide antenna which would only allow the fundamental E^y_{11} mode to propagate at 94 GHz for silicon ($\varepsilon_r = 12$), sapphire ($\varepsilon_r = 9.4$), and boron nitride ($\varepsilon_r = 4.0$) material. For any given frequency, the smaller the difference between the relative dielectric constant of the waveguide and the dielectric constant of the surrounding medium (usually air) the larger the guide can be. The guide size (height and width) at a given frequency determines the guide wavelength, which is a key parameter for determining the angle at which the energy will radiate from the antenna. Theoretical calculations were made to establish the angles of radiation at frequencies between 86 and 94 GHz for silicon, sapphire, and boron nitride waveguide antennas having the theoretical maximum allowed dimensions for the fundamental E^y_{11} mode. The effect on the radiation angles due to changes in waveguide size and perturbation spacing was determined.

---

1. K. L. Klohn, R. E. Horn, H. Jacobs, E. Freibergs, "Silicon Waveguide Frequency Scanning Linear Array Antenna," IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-26, Nr. 10, October 1978.
2. T. Itoh, "Leaky-Wave Antenna and Band Reject Filter for Millimeter-Wave Integrated Circuits," 1977 IEEE MTT-S International Microwave Symposium Digest, June 21-23, 1977, pp 538-541.
3. N. Williams, A. W. Rudge and S. E. Gibbs, "Millimeter-Wave Insular Guide Frequency Scanned Array," 1977 IEEE MTT-S International Microwave Symposium Digest, June 21-23, 1977, pp 542-544.
4. E. A. J. Marcatili, "Dielectric Rectangular Waveguide and Directional Coupler for Integrated Optics," Bell System Technical Journal, Vol, 48, No. 7, September 1969, pp 2071-2102.

CALCULATIONS

Equation (1) was used to find a good approximation for the maximum allowable guide size to maintain single E^y_{11} mode operation at 94 GHz for silicon, sapphire, and boron nitride.

$$k_z = \sqrt{k_1^2 - k_x^2 - k_y^2} \qquad (1)$$

where k_z = propagation constant down the guide, z - direction

k_x, k_y = transverse propagation constants, x- and y- directions respectively

k_1 = propagation constant in the dielectric guide.

The propagation constants were calculated from Marcatili's equations[5] as indicated in the Appendix. As long as k_z is real for any given mode, $(k_1^2 > k_x^2 + k_y^2)$, that mode can propagate. Since k_x is independent of the b-dimension and k_y is independent of the a-dimension, the maximum a-dimension will be the size at which the next higher mode in the x-direction E^y_{21} begins to propagate and the maximum b-dimension will be the size at which the next higher mode in the y-direction E^y_{12} begins to propagate. The results of the theoretical calculations are given in Figures 1, 2 and 3 and summarized in Table I.

TABLE I

MAXIMUM ALLOWED GUIDE SIZE FOR SINGLE E^y_{11} MODE AT 94 GHz

MATERIAL	ϵ_r	WIDTH a(mm)	HEIGHT b(mm)	λ_g(mm)
Silicon	12	1.0	0.9	1.2
Sapphire	9.4	1.2	1.0	1.3
Boron Nitride	4	2.0	1.5	1.9

Once the a- and b- dimensions of the antenna are determined, the transverse propagation constants will be fixed and guide wavelength λ_g can be calculated from Equation (2).

$$\lambda_g \equiv \lambda_z = 2\pi / k_z \qquad (2)$$

5. Ibid 4

With the geometry fixed, λ_g can be varied by changing the input frequency. This change in λ_g is illustrated in Figure 4 (solid curve) for frequencies between 86 and 94 GHz. The guide sizes of the three dielectric materials investigated were the maximum allowable for E^y_{11} mode operation at 94 GHz (Table I).

The angles of radiation for the frequencies between 86 and 94 GHz for silicon, sapphire and boron nitride were calculated using the following equation.[6,7]

$$\theta_n = \sin^{-1}\left(\frac{\lambda_o}{\lambda_g} + \frac{\lambda_o}{d} n\right) \tag{3}$$

where $\left|\frac{\lambda_o}{\lambda_g} + \frac{\lambda_o}{d} n\right| \leq 1$

θ_n = beam angle from broadside (normal) for n^{th} space harmonic

λ_o = free space wavelength

d = perturbation spacing

n = space harmonic; $0, \pm 1, \pm 2 \ldots$

Figure 5 illustrates the range of angular scan for the 8 GHz frequency variation for the three dielectrics having the following corresponding slopes: silicon, 2.8 deg/GHz, sapphire, 2.6 deg/GHz and boron nitride, 1.6 deg/GHz.

DISCUSSION

Using Marcatili's equations as a basis, it was noted that for any given frequency of operation, a dielectric guide in air can be made larger as the ε_r of the guide material becomes smaller. The theoretical curves in Figure 6 present the data for maximum waveguide dimensions at unity aspect ratio (a/b=1) for frequencies between 60 and 220 GHz and materials with relative dielectric constants ranging from 2 to 12. The curves show that as ε_r increases there is less and less change in the allowable maximum waveguide dimensions. In addition, this flattening of the curve occurs at lower ε_r values as the frequency increases. Thus, at 220 GHz there is no change in maximum allowed guide size between Si (ε_r = 12) and sapphire (ε_r = 9.4). As ε_r decreases to lower values (< 8) the increase in allowable guide size can be quite dramatic. Figures 7 and 8 indicate the relative sizes of different dielectric antenna structures at 94 GHz and 220 GHz respectively.

6. A. A. Oliner, Informal Communication, Class Notes from Polytechnical Institute of New York.
7. A. Hessel, "General Characteristics of Traveling-Wave Antennas," Antenna Theory Part 2, Collins & Zucker, McGraw-Hill Book Company, New York, NY 1969.

The size increase which results from going to dielectric with smaller ϵ_r can be quite advantageous since it will ease fabrication problems and handling. When cutting a 10 cm long dielectric bar from a sheet of material, the bar width (waveguide a-dimension) typically varied by approximately 0.1 mm. At 94 GHz, this represented a 10% change in the a-dimension for Si but only a 5% change for BN. The effect this has on λ_g is indicated by the dashed curves in Figure 4. For Si, the change in λ_g was 1.6% at 94 GHz and 1.9% at 86 GHz, whereas, for BN, the changes were only 0.6% and 0.7% for 94 and 86 GHz respectively. The changes in λ_g in turn, translated into angular changes of approximately 3^o for Si, but only 0.6^o for BN. The data for Si, sapphire, and BN is tabulated in Table II.

TABLE II

EFFECT OF 0.1 mm DECREASE IN MAXIMUM 94 GHz a-DIMENSION

Material	f(GHz)	a max (mm)	Δa(%)	$\Delta\lambda_g$(%)	$\Delta\theta$(deg)
Si	94	1.0	-10	+1.6	-2.6
	86	1.0	-10	+1.9	-3.0
Sapphire	94	1.2	-8.3	+1.3	-1.8
	86	1.2	-8.3	+1.5	-2.2
BN	94	2.0	-5	+0.6	-0.5
	86	2.0	-5	+0.7	-0.6

The effect became more pronounced as frequency increased. Table III summarizes the effect at 220 GHz for the same cutting tolerance of 0.1 mm in the a-dimension for Si and BN.

TABLE III

EFFECT OF 0.1 mm DECREASE IN MAXIMUM 220 GHz a-DIMENSION

Material	a max (mm)	Δa(%)	$\Delta\lambda_g$(%)	$\Delta\theta$(deg)
Si	0.4	-25	+5.5	-8.8
BN	0.6	-16.7	+3.7	-3.1

Eliminating dimensional variations due to normal cutting tolerances would require additional polishing which, in turn, would add to the fabrication costs. Since this type of antenna would potentially find high volume use in expendable applications, cost is a very important consideration.

Examination of the E^y_{11} field distributions for Si and BN at 94 GHz plotted in Figure 9 revealed the fact that the electric field in BN at the point where it is 1/e of its maximum value, extends twice as far ($\eta_{2,4}$ = 0.35 mm, $\xi_{3,5}$ = 0.31 mm) into the surrounding medium (air) as it does in the case of Si. ($\eta_{2,4}$ = 0.18 mm, $\xi_{3,5}$ = 0.16 mm). This will necessitate some extra precautions in mounting, such that any nearby metal does not disrupt the field causing an unwanted change in guide wavelength.

Any variation, such as physical size, frequency, or relative dielectric constant, which changes λ_g will result in a change in the radiation angle. The effect of changing λ_g by $\pm$ 10% in each of the three dielectrics, Si, sapphire, and BN is illustrated in Figure 10. The results showed that the higher ϵ_r material will exhibit a greater angular variation for a given $\Delta\lambda_g$; i.e. for Si, 143°/mm $\Delta\lambda_g$, sapphire, 104°/mm $\Delta\lambda_g$, and BN, 47°/mm $\Delta\lambda_g$.

The spacing of the metal perturbations also has a great deal of influence on the angle of radiation. Changes in spacing of $\pm$ 0.1 mm can result in a substantial shift in the angle of radiation. The results of radiation angle calculations for various perturbation spacings and physical sizes of the guide are tabulated in Table IV.

TABLE IV

RADIATION ANGLES FOR VARIOUS a-DIMENSIONS AND PERTURBATION SPACINGS AT 94 GHz

Material	a(mm)	b(mm)	d(mm)	λ_g(mm)	θ(deg)
Si	1.0	0.9	1.2	1.159	5.4
	1.0	0.9	1.3	1.159	17.4
	1.0	0.9	1.1	1.159	-8.5
	0.9	0.9	1.2	1.178	2.8
	0.9	0.9	1.3	1.178	14.7
	0.9	0.9	1.1	1.178	-11.1
Sapphire	1.2	1.0	1.3	1.300	0.0
	1.2	1.0	1.4	1.300	10.1
	1.2	1.0	1.2	1.300	-11.8
	1.1	1.0	1.3	1.317	-1.8
	1.1	1.0	1.4	1.317	8.3
	1.1	1.0	1.2	1.317	-13.7
BN	2.0	1.5	1.9	1.936	-1.8
	2.0	1.5	2.0	1.936	3.0
	2.0	1.5	1.8	1.936	-7.2
	1.9	1.5	1.9	1.947	-2.3
	1.9	1.5	2.0	1.947	2.5
	1.9	1.5	1.8	1.947	-7.7

In the case of Si, the shift in radiation angle due to a change of 0.2 mm in the spacing of the metal stripe perturbations was about 25°. In the extreme case where the guide's a-dimension also changes by as much as 0.1 mm, an additional 3° angular shift will occur, giving a $\Delta\theta$ in the order of 28°. In BN the angular shift, although still significant, was less than half of that theoretically calculated for Si. The results indicate a $\Delta\theta$ for BN of approximately 10° due to a 0.2 mm change in d together with a 0.1 mm change in the a-dimension.

SUMMARY

Theoretical calculations were made to determine the effect of using various dielectric materials for millimeter-wave antennas. The results of these calculations indicated that materials with a low ϵ_r can be made substantially larger in cross sectional area than high ϵ_r dielectrics. As operating frequency increases, however, this advantage becomes less pronounced. Larger waveguides have the advantages of being easier to handle, less fragile, and have a lower percentage dimensional variation occurring in normal fabrication. The last factor helps to achieve repeatable performance while reducing fabrication cost because of the reduction or elimination of the need for precision machining or additional polishing.

At 94 GHz, a BN (ϵ_r = 4.0) antenna can be made with a cross sectional area approximately three times larger than Si (ϵ_r = 12). A cutting tolerance of 0.1 mm on the a-dimension of the guide was only 5% in the case of BN which translated into ~ 0.5° change in radiation angle whereas this same 0.1 mm dimensional change in Si was a 10% variation and caused an ~ 3° angular change; a six-fold increase. The shifts in radiation angle resulting from a change in perturbation spacing as indicated in Table IV, clearly points out the necessity of maintaining close control on the metal stripe spacing. A great deal of uniformity would result from utilizing a precision mask and evaporating the metal stripes onto the surface of the dielectric guide.

A disadvantage of low ϵ_r material is illustrated in Figures 5 and 10. In order to achieve the same number of degrees of angular scan for a frequency scanning antenna, a larger Δf_o was required for BN (1.6 degrees/GHz) than for Si (2.8 degrees/GHz). When the more fundamental guide wavelength was examined (Figure 10), which can be varied by changing either the frequency, relative dielectric constant, or physical dimensions, it was again noted that a larger $\Delta\lambda_g$ was required for BN (47 degrees/mm $\Delta\lambda_g$) than for Si (143 degrees /mm $\Delta\lambda_g$) to achieve the same number of degrees of angular scan.

Another consideration taken into account was the advantage of using a semiconductor dielectric such as Si in which a millimeter-wave source can be grown in-situ. This can result in substantial cost savings in the overall subsystem by using a batch type of processing. Achieving low cost is one of the primary considerations for the potential use of dielectric line scanning antennas in expendable applications such as projectile and missile terminal homing.

ACKNOWLEDGEMENT

The author would like to thank Dr. Harold Jacobs and Robert E. Horn of the Millimeter Wave Devices and Circuits Team, Microwave and Signal Processing Devices Division, Electronics Technology and Devices Laboratory, ERADCOM, for their contributions. Dr. Harold Jacobs for his stimulating discussions and encouragement and Robert E. Horn for discussions relating to his experimental work.

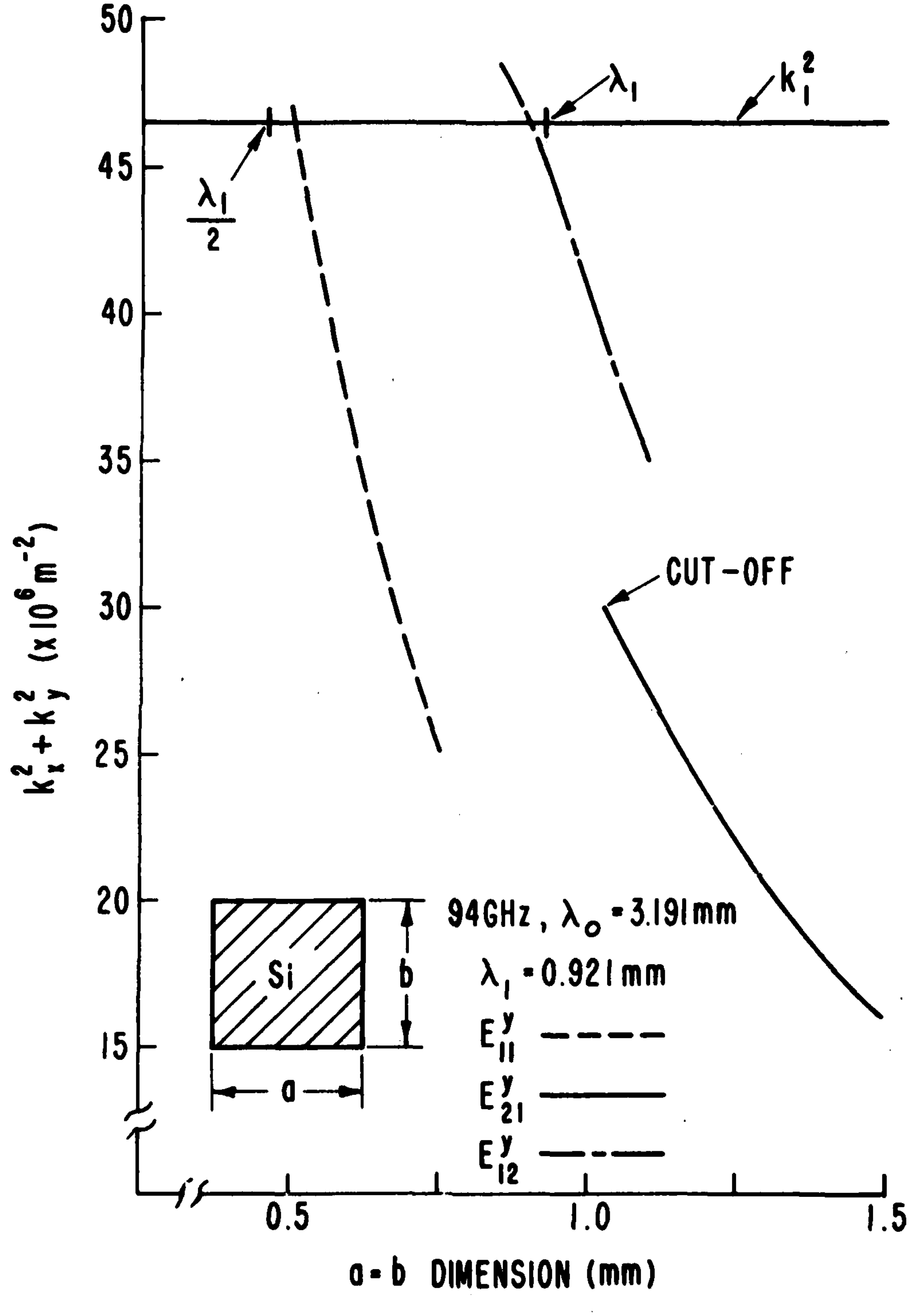

FIGURE 1. MINIMUM AND MAXIMUM SILICON WAVEGUIDE DIMENSIONS, 94 GHz

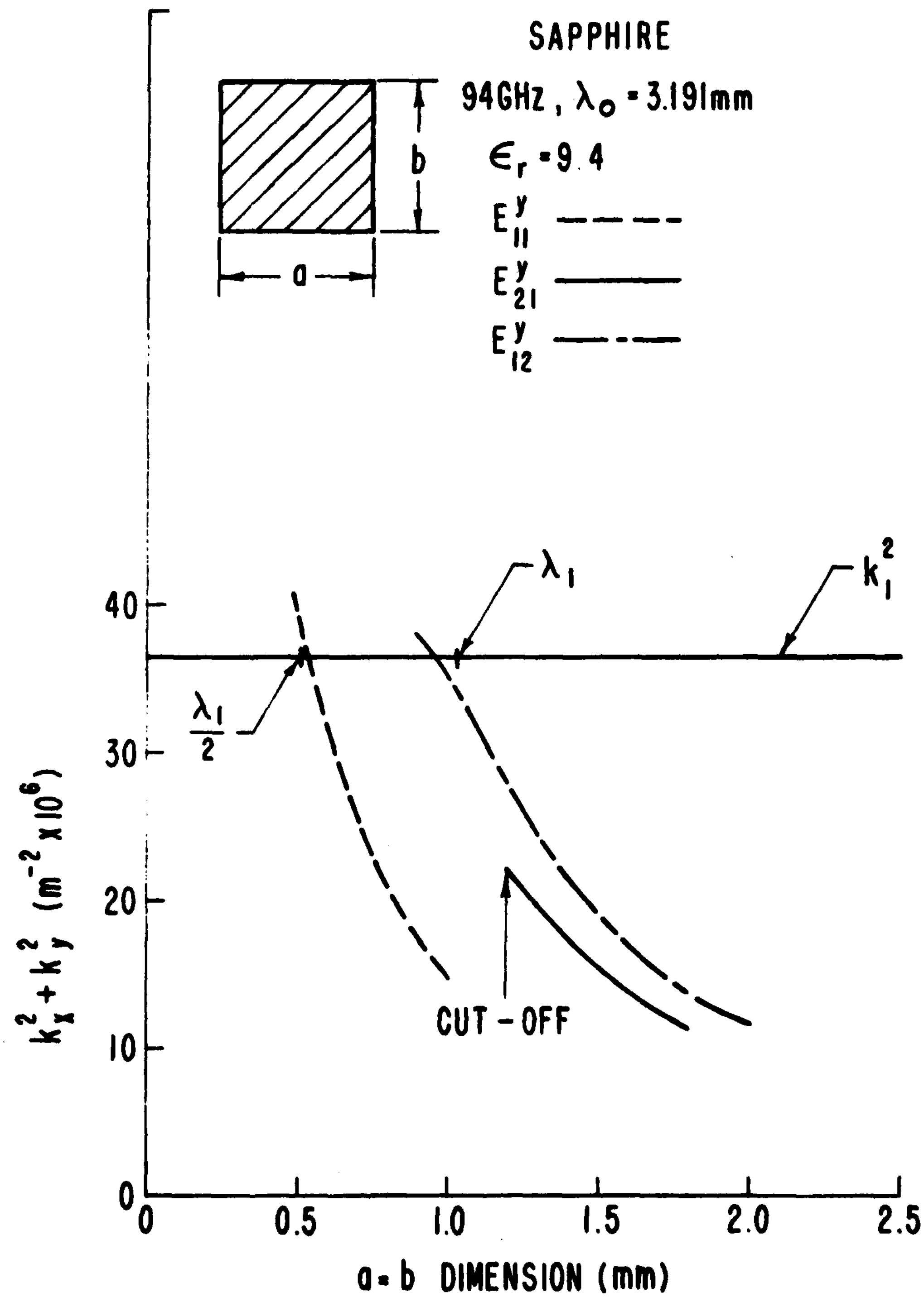

FIGURE 2. MINIMUM AND MAXIMUM SAPPHIRE WAVEGUIDE DIMENSIONS, 94 GHz

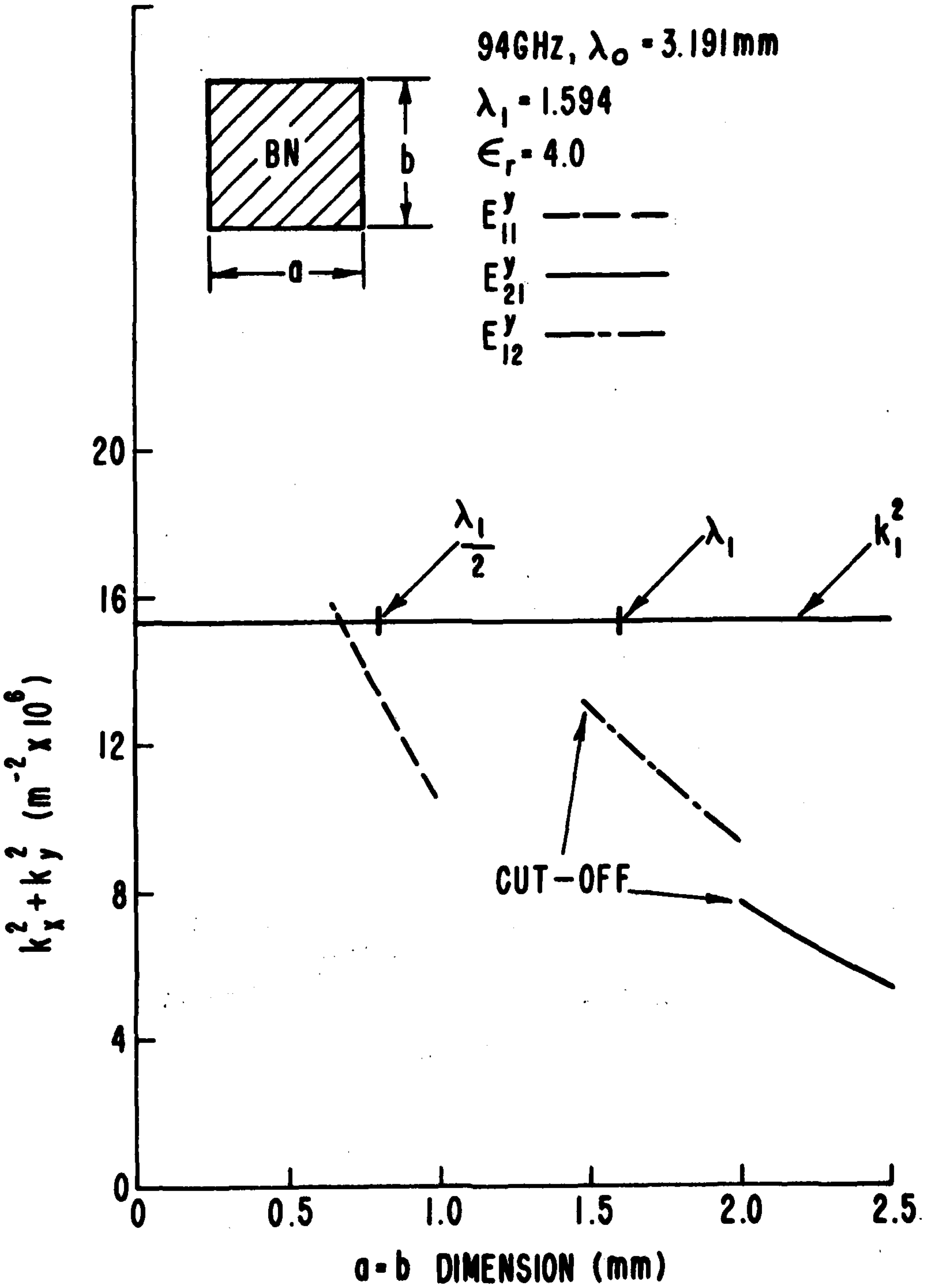

FIGURE 3. MINIMUM AND MAXIMUM BORON NITRIDE WAVEGUIDE DIMENSIONS, 94 GHz

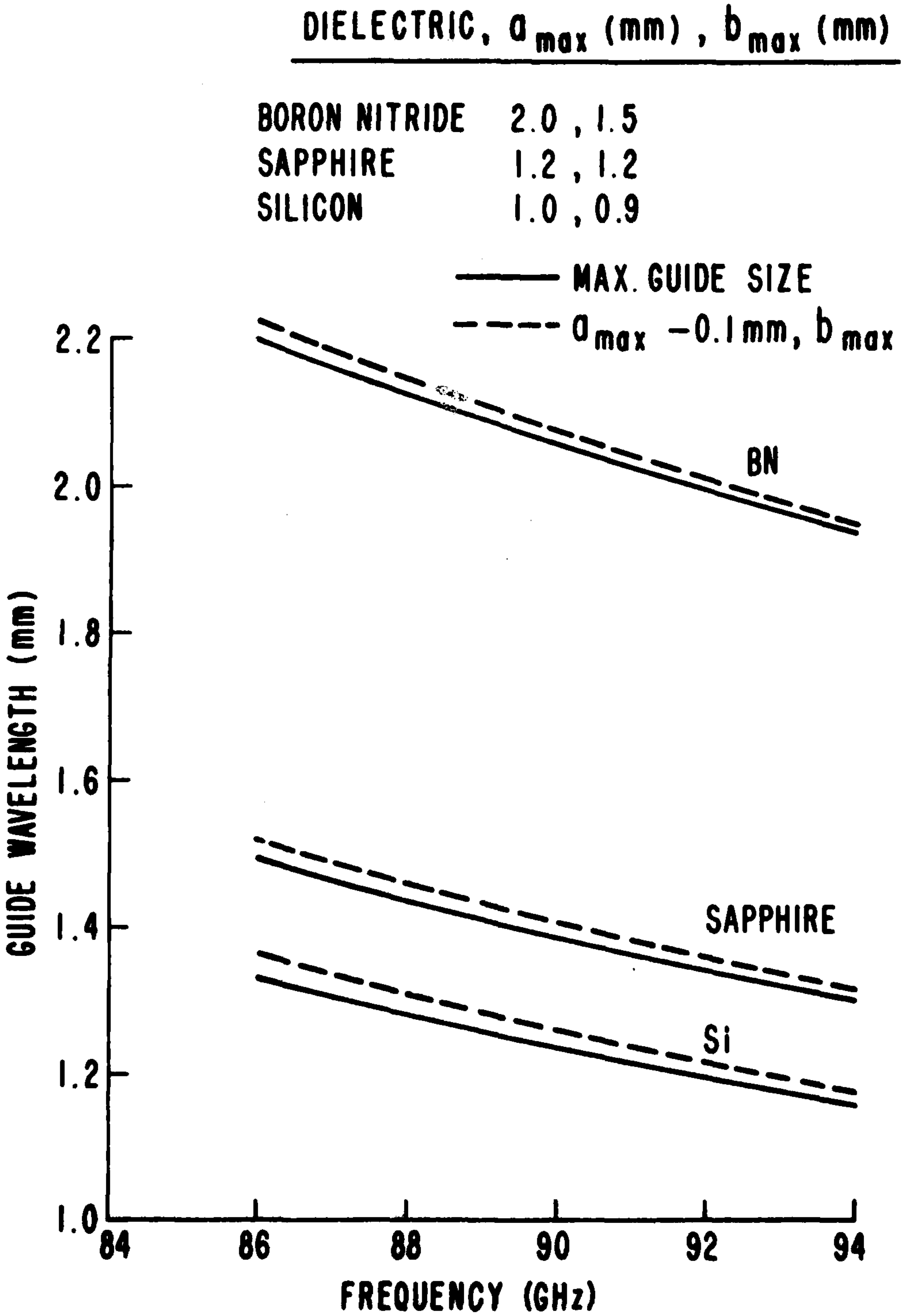

FIGURE 4. VARIATION OF GUIDE WAVELENGTH VERSUS FREQUENCY

	a (mm)	b (mm)	d (mm)
SILICON	1.0	0.9	1.2
SAPPHIRE	1.2	1.0	1.3
BORON NITRIDE	2.0	1.5	1.9

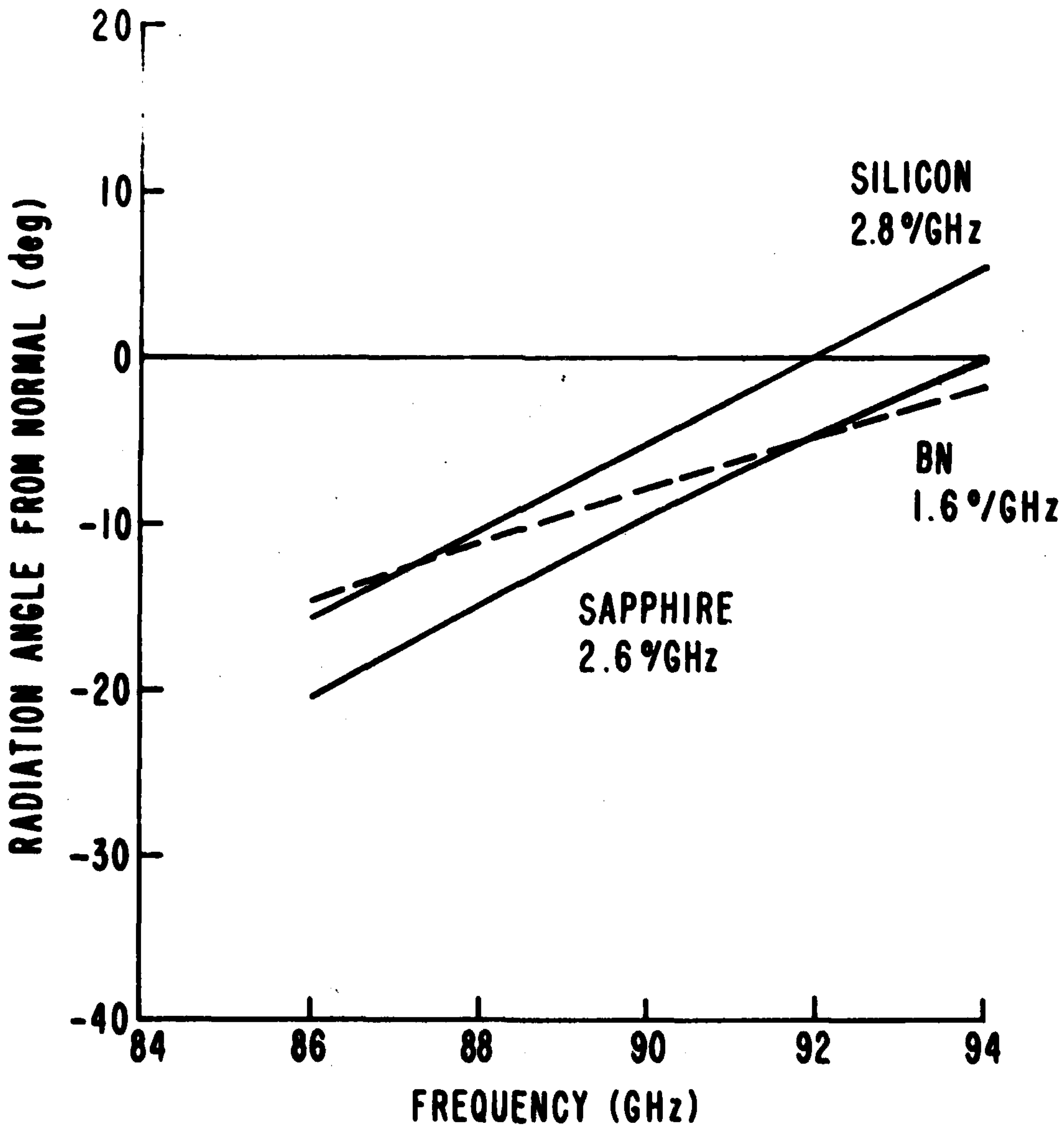

FIGURE 5. RADIATION ANGLE FROM NORMAL VERSUS FREQUENCY

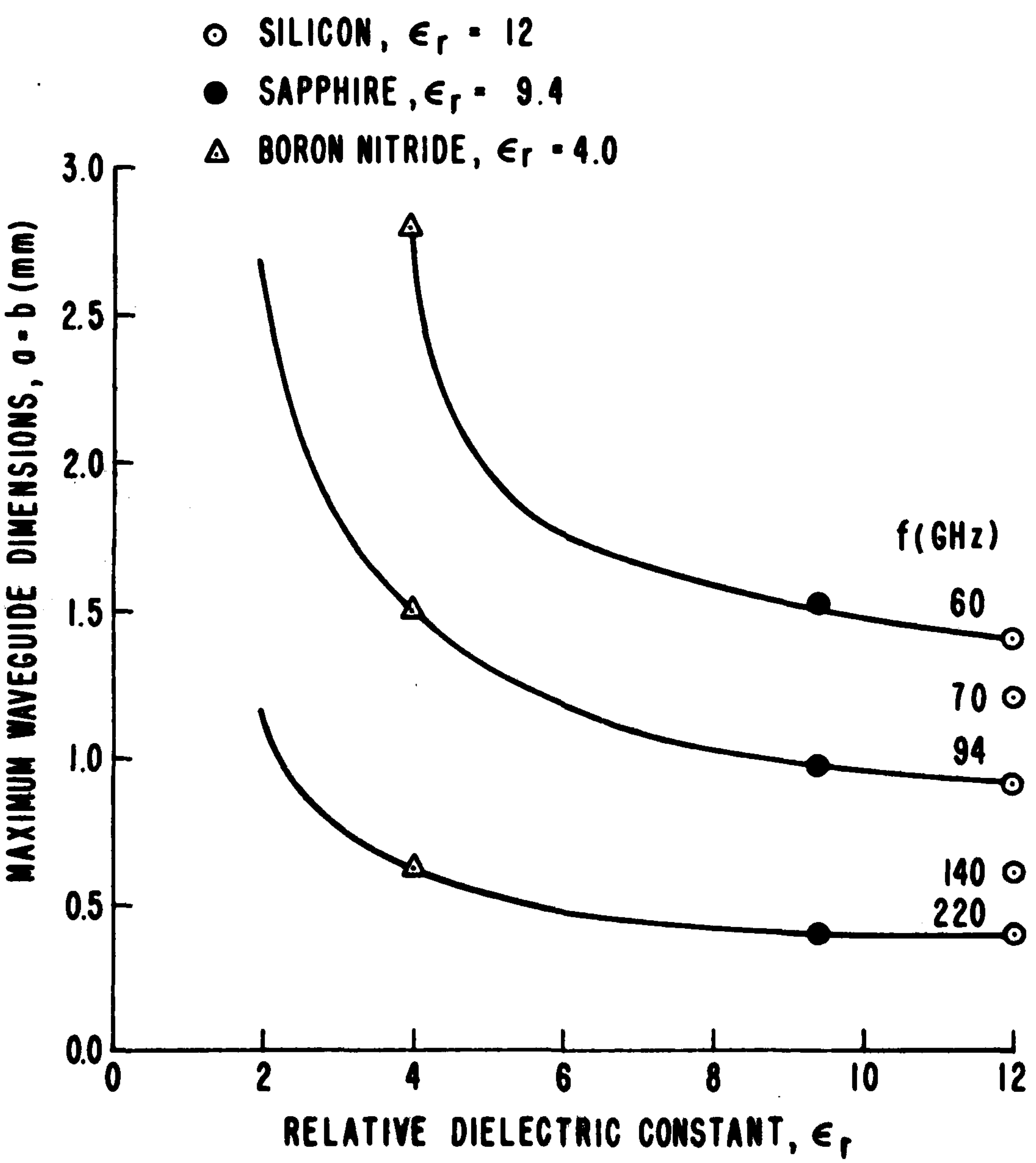

FIGURE 6. MAXIMUM ALLOWABLE WAVEGUIDE DIMENSIONS VERSUS RELATIVE DIELECTRIC CONSTANT

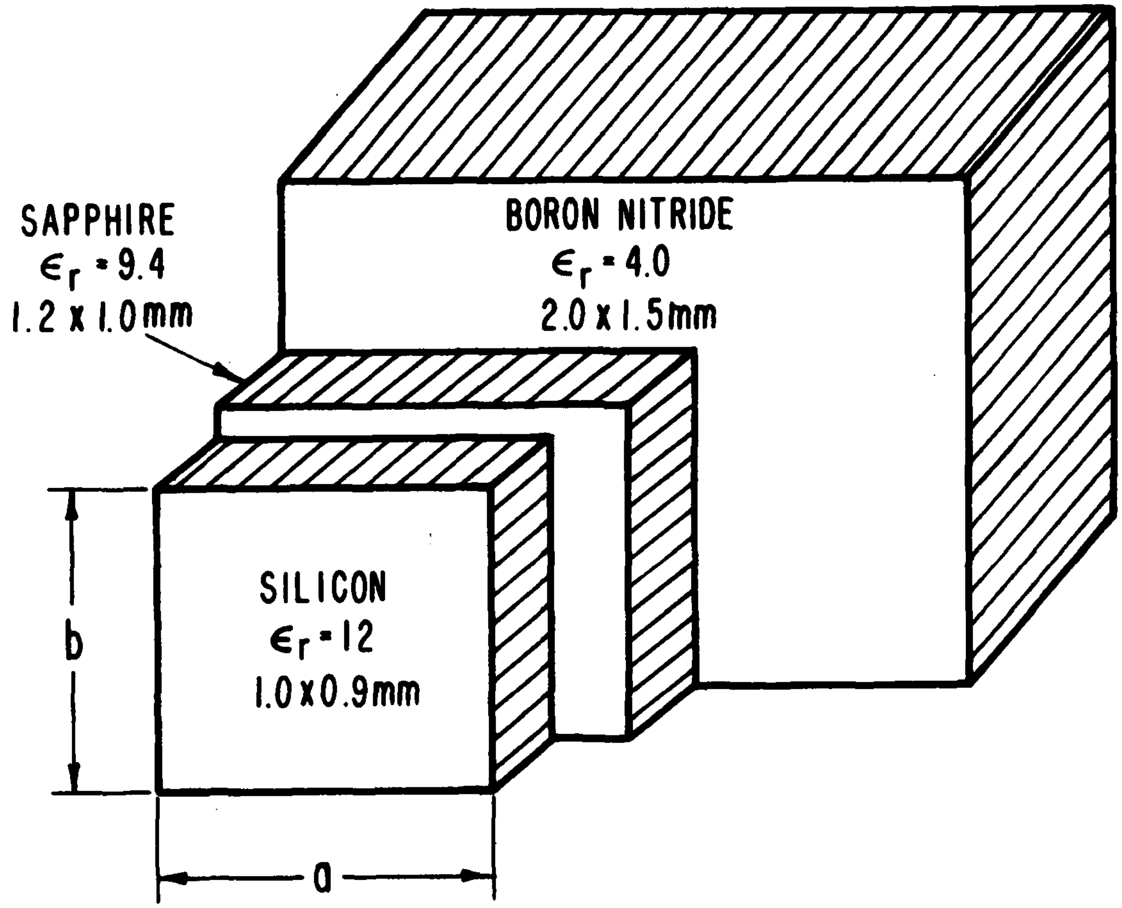

FIGURE 7. RELATIVE MAXIMUM GUIDE SIZES AT 94 GHz

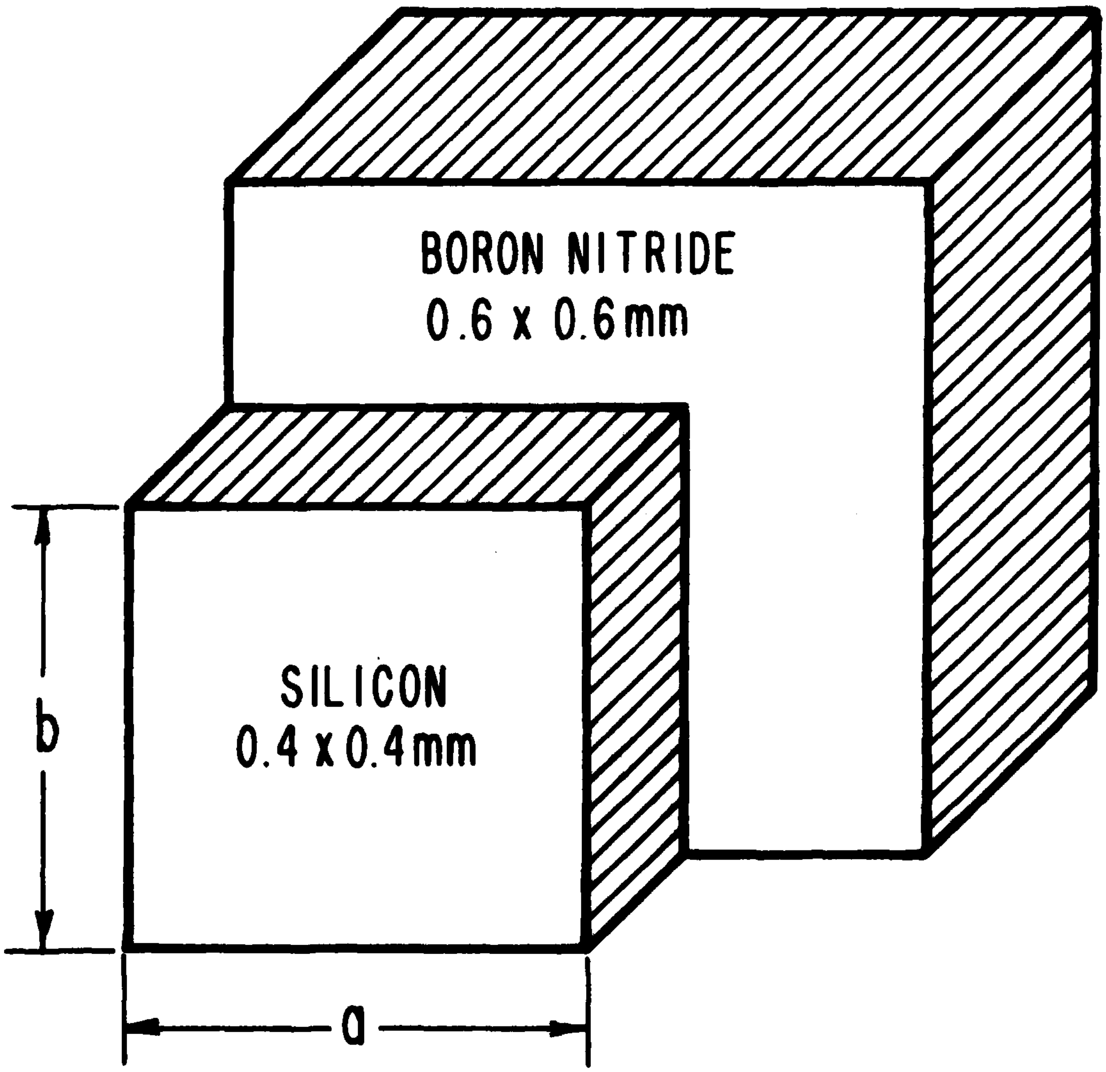

FIGURE 8. RELATIVE GUIDE SIZES FOR UNITY ASPECT RATIO AT 220 GHz

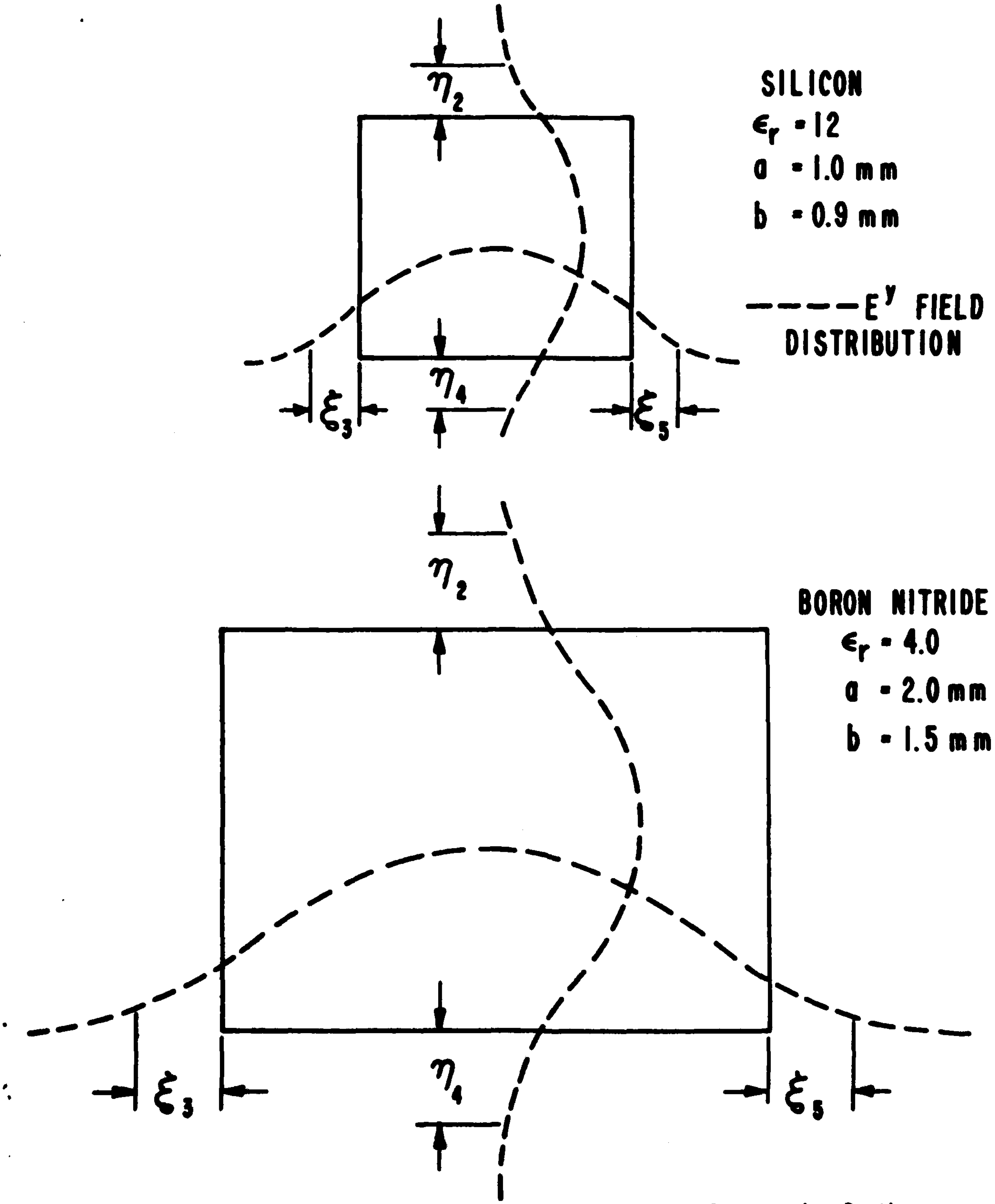

FIGURE 9. Relative Field Distribution and Penetration Depths for Silicon and Boron Nitride Guides at 94 GHz

$94\ \mathrm{GHz},\ \lambda_o = 3.191\ \mathrm{mm}$

DIELECTRIC	a (mm)	b (mm)	d (mm)
SILICON	1.0	0.9	1.2
SAPPHIRE	1.2	1.0	1.3
BORON NITRIDE	2.0	1.5	1.9

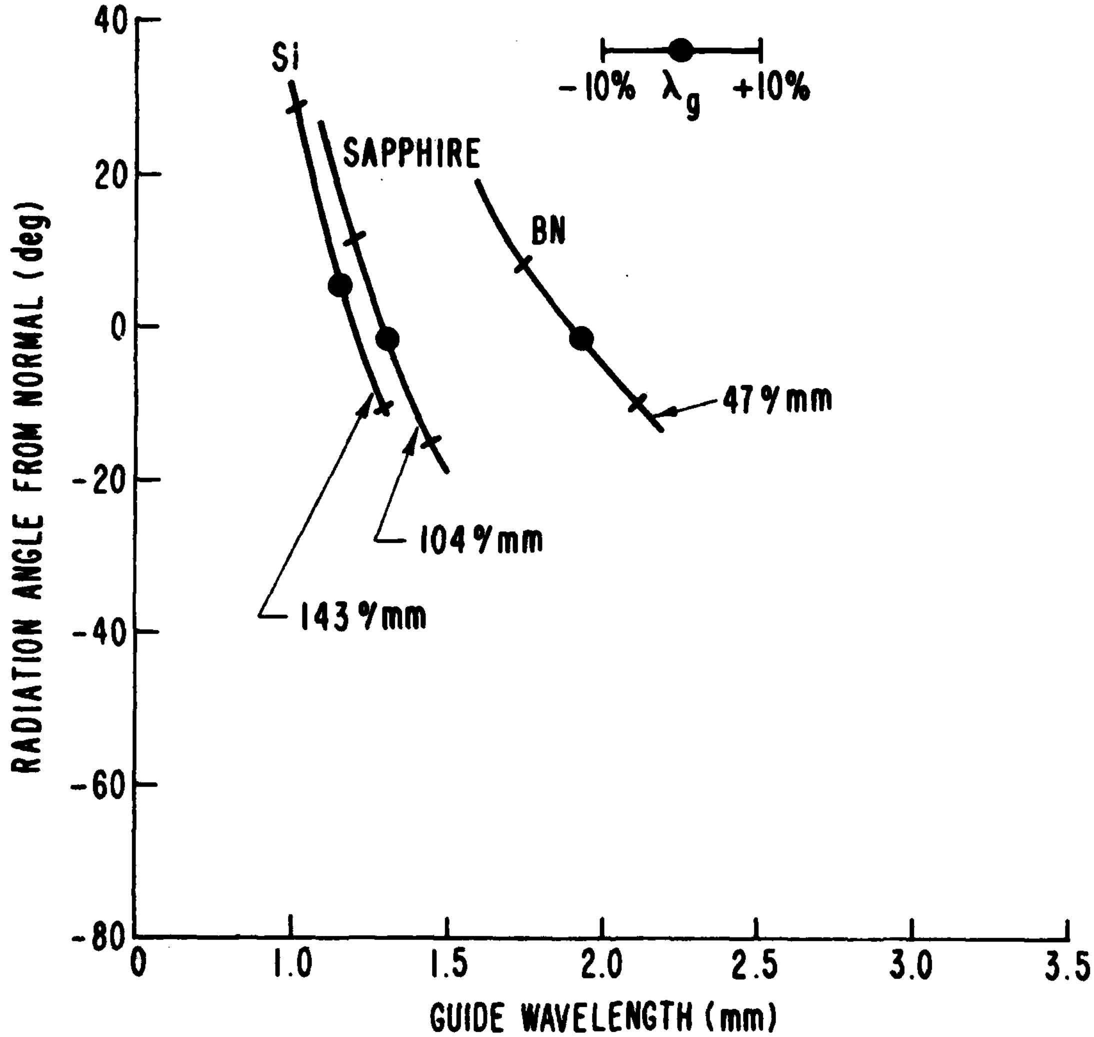

FIGURE 10. RANGE OF RADIATION ANGLE FOR CHANGES IN GUIDE WAVELENGTH AT 94 GHz

APPENDIX

CALCULATION OF PROPAGATION CONSTANTS AND RADIATION ANGLES

Marcatili[1] developed the following basic equations for calculating the propagation constants in dielectric waveguides:

$$k_1 = 2\pi n_1 / \lambda_0 \tag{A1}$$

and

$$k_z = \sqrt{k_1^2 - k_x^2 - k_y^2} \tag{A2}$$

The transverse propagation constants k_x and k_y are solutions of the transcentental equations

$$k_x a = p\pi - \tan^{-1}(k_x \xi_3) - \tan^{-1}(k_x \xi_5) \tag{A3}$$

$$k_y b = q\pi - \tan^{-1}\left(\frac{n_2^2}{n_1^2} k_y \eta_2\right) - \tan^{-1}\left(\frac{n_4^2}{n_1^2} k_y \eta_4\right) \tag{A4}$$

where a and b are the width and height of the guide respectively. The number of extrema p, in the x-direction, and q, in the y-direction equal one for the E^y_{11} mode. The lengths $\xi_{3,5}$ and $\eta_{2,4}$ indicate the distances the electric fields penetrate the respective surrounding mediums until the field amplitude has decayed to 1/e of the maximum field in the respective medium

where

$$\xi_{3,5} = \frac{1}{\sqrt{\left(\frac{\pi}{A_{3,5}}\right)^2 - k_x^2}} \tag{A5}$$

$$\eta_{2,4} = \frac{1}{\sqrt{\left(\frac{\pi}{A_{2,4}}\right)^2 - k_y^2}} \tag{A6}$$

$A_{2,3,4,5}$ indicates the maximum physical dimensions of the waveguide which will support only the fundamental mode provided the guide is surrounded by a uniform medium of equal refractive index.

$$A_{2,3,4,5} = \frac{\lambda_0}{2\sqrt{n_1^2 - n_{2,3,4,5}^2}} \tag{A7}$$

1. E. A. J. Marcatili, "Dielectric Rectangular Waveguide and Directional Coupler for Integrated Optics," Bell System Technical Journal, Vol 48, No. 7, Sep 69.

Equations (A3) and (A4) were solved on a programmed calculator by solving and plotting the right and left hand sides of each equation for a range of k_x and k_y values respectively. By employing the "course grid approach" to determine the point of intersection of the two curves for each equation, the exact numerical values of k_x and k_y were determined[2].

TABLE A-I

PROPAGATION CONSTANTS AND PENETRATION DEPTHS

E^y_{11} Mode, f = 94 GHz, λ_o = 3.191 mm, $n_2 = n_4 = 1.0$

Dielectric	n_1	a,b (mm)	k_x (m^{-1})	k_y (m^{-1})	$\eta_{2,4}$ (mm)	$\xi_{3,5}$ (mm)
Silicon	3.464	1.0, 0.9	2392	3379	0.18	0.16
Sapphire	3.006	1.2, 1.0	2016	3010	0.21	0.19
Boron Nitride	2.0	2.0, 1.5	1209	1877	0.35	0.31

As an example, for silicon, Equation (A3) becomes:

$$2392\ m^{-1}(1.0\times10^{-3}\ m) = (1)\pi - 2\tan^{-1}(2392\ m^{-1}\cdot 0.16\times10^{-3}\ m)$$

$$2.392 = \pi - 2\tan^{-1}(0.383)$$

since $\tan^{-1}(0.383) = 20.96^\circ = 0.366$ radians

then $2.392 \simeq 3.142 - 0.732 = 2.410$

Equation (A4) becomes:

$$3379\ m^{-1}(0.9\times10^{-3}\ m) = (1)\pi - 2\tan^{-1}\left(\tfrac{1}{12}\cdot 3379\ m^{-1}\cdot 0.18\times10^{-3}\ m\right)$$

$$3.041 = \pi - 2\tan^{-1}(0.0507)$$

since $\tan^{-1}(0.0507) = 2.902^\circ = 0.0506$ radians

then $3.041 \simeq 3.142 - 0.102 = 3.040$

2. K. L. Klohn, J. F. Armata, Jr., and M. M. Chrepta, "Transverse Propagation Constants in Dielectric Waveguides," R&D Technical Report ECOM 4242, US Army Electronics Command, Fort Monmouth, NJ, Aug 74.

Using Equations (A1) and (A2) for Si at 94 Ghz, k_1 and k_z were calculated.

$$k_1 = \frac{2\pi}{3.191 \times 10^{-3}\,m}(3.464) = 6823\ m^{-1}$$

$$k_z = \sqrt{(6823\ m^{-1})^2 - (2392\ m^{-1})^2 - (3379\ m^{-1})^2}$$

$$k_z = 5423\ m^{-1}$$

The guide wavelength was then calculated using

$$\lambda_g \equiv \lambda_z = 2\pi / k_z \tag{A8}$$

For Si at 94 GHz, the resutl is

$$\lambda_z = 2\pi / 5423\ m^{-1} = 1.159\ mm$$

Results of the calculations are in TABLE A-II

TABLE A-II

PROPAGATION CONSTANTS AND GUIDE WAVELENGTH, E^y_{11} MODE

$f = 94$ GHz, $\lambda_o = 3.191$ mm

Dielectric	a,b (mm)	k_1 (m^{-1})	k_z (m^{-1})	λ_z (mm)
Si	1.0, 0.9	6823	5423	1.159
Sapphire	1.2, 1.0	6039	4832	1.344
BN	2.0, 1.5	3939	3242	1.936

RADIATION ANGLE FROM NORMAL

The angles of radiation were calculated using[3]

$$\theta_n = \sin^{-1}\left(\frac{\lambda_o}{\lambda_g} + \frac{\lambda_o}{d} n\right) \tag{A9}$$

3. A. A. Oliner, Informal Communication, Class Notes from Polytechnical Institute of New York.

For a given free space wavelength λ_0, guide wavelength λ_g, n = -1 space harmonic, the radiation agnle θ_n can vary substantially by changing the perturbation spacing d.

As an example, to determine θ_n for a Si guide with a = 1.0 mm, b = 0.9 mm at 94 GHz (λ_0 = 3.191 mm), and d = 1.1 mm, Equation (A10)

gave $$\theta_n = \sin^{-1}\left[\frac{3.191}{1.159} + \frac{3.191}{1.1}(-1)\right] = -8.493^\circ$$

If the perturbation spacing is changed to 1.2 mm, Equation (A10)

becomes $$\theta_n = \sin^{-1}\left[\frac{3.191}{1.159} + \frac{3.191}{1.2}(-1)\right] = 5.399^\circ$$

This is nearly a 13° change in radiation angle caused by only a 0.1 mm change in d.